Low Complexity Frequency Domain TOA Estimation for IR-UWB Communications

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Abstract—This paper addresses the problem of TOA estimation for ranging applications in IR-UWB. It considers a low complexity frequency-domain approach for the estimation of the TOA that allows sub-Nyquist sampling rate receivers. Two diversity schemes are investigated in combination with the TOA estimation algorithm and are evaluated over realistic channel models, yielding accurate ranging estimation.

I. INTRODUCTION

Communication systems based on impulse radio ultra-wideband (IR-UWB) have been envisaged as radio communication systems that could enable very accurate ranging and location applications, given the extremely short duration pulses. This high time resolution nature of the UWB signal makes TOA estimation method a good candidate for positioning estimation in UWB communications.

Ranging accuracy depends on how precisely the receiver can discriminate the first arriving signal, which in a multipath environment may not be the strongest. In the literature most of ranging techniques are based on time-domain TOA estimation methods. The maximum likelihood (ML) solution has practical limitations due to the requirement of very high sampling rates. Typically, the conventional correlation-based approach results in a very slow TOA estimator [1] requiring an exhaustive search over a large number of beams. Iterative ML approaches have been also studied [2] but yet requiring very high rate sampling.

Recently, it has appeared in the literature proposals to reduce the sampling constraints and time intervals required for estimation of time-domain based TOA estimators. An approach addressed by several authors consist on a two step TOA estimation process consisting of an initial coarse estimation of the TOA followed by a higher resolution stage. Following this strategy, the authors in [3] propose a first rough TOA estimate based on the received signal energy, followed by a low-rate correlation stage that estimates the TOA based on hypothesis testing. In [4] a similar two step estimator is proposed based on a threshold based energy detection receiver. The scheme allows for a symbol rate sampling but requires using several symbols and an appropriate design of the signal waveform. Two stage approach is also considered in [5] were a low cost non-coherent receiver based on an energy detection stage is proposed. The basic principle is based on integration windows which time resolution changes between the two stages. A critical parameter for these type of estimators lies on the threshold selection.

Motivated by recent work on UWB receiver architectures that provides direct samples of the received signal in the frequency domain at sub-Nyquist sampling rate [6] [7], in this paper we propose the use of low complexity frequency-based TOA estimation techniques. The use of frequency domain TOA estimation methods has recently been considered to provide high resolution estimates for positioning applications in UMTS framework [8]. In [9] the authors also examined frequency domain high-resolution methods for TOA estimation for indoor geolocation. The aim of this paper is to provide a low complexity TOA estimation method requiring a sub-Nyquist sampling rate.

The rest of the paper is organized as follows. The signal model is presented in Section II. Section III introduces the frequency-domain TOA estimation algorithm followed by the proposed ranging technique in Section IV. Evaluation and performance results are detailed in Section V. Finally, conclusions are drawn in Section VI.

II. UWB SIGNAL MODEL

We considered an IR-UWB system where transmission of an information symbol is typically implemented by the repetition of $N_p$ pulses of very short duration,$$
s(t) = \sum_{p=0}^{N_p-1} \sqrt{E_p} p(t - pT_{pr})
$$
where $E_p$ denotes the energy of one pulse, $p(t)$ refers to the single pulse waveform, typically a Gaussian monocycle or its derivatives $^1$, and $T_{pr}$ is the repetition pulse period.

$^1$Modulation and time hopping can be explicitly include in the pulse waveform such as $p(t) = p_c(t - b_i T_c) - c_i T_c$ with $p_c(t)$ being typically a Gaussian monocycle or its derivatives, $b_i$ the information symbol, $T_c$ modulation time shift, $c_i$ the time hopping sequence and $T_c$ the chip interval. We assume symbol synchronization is available at the receiver (eg: via transmission of pilot symbols)
A typical model for the multipath fading channel is given by,

$$h(t) = \sum_{m=1}^{M} h_m \delta(t - \tau_m)$$

channel impulse response, where \( h_m \) denotes the fading coefficient for the \( m \)-th path while \( \tau_m \) represents the delay experienced by the \( m \)-th path. The received signal is then the summation of multiple delayed and attenuated replicas of the transmitted signal,

$$y(t) = \sum_{m=1}^{M} h_m s(t - \tau_m) + v(t)$$

where \( v(t) \) denotes the contribution of the additive Gaussian noise. Transforming the signal to the frequency domain,

$$Y(w) = \sum_{m=1}^{M} h_m S(w)e^{-j\omega \tau_m} + V(w)$$

where \( Y(w) \), \( S(w) \) and \( V(w) \) denote the Fourier transform of the received signal, transmitted signal and noise, respectively.

III. FREQUENCY DOMAIN TOA ESTIMATION

Rearranging (2) into a matrix notation the received signal is given by,

$$\mathbf{Y} = \mathbf{SE}_r \mathbf{h} + \mathbf{V}$$

where the elements of vector \( \mathbf{Y} \in \mathbb{C}^{N \times 1} \), \( Y(w_k) \), are the DFT components of \( y(t) \) with \( w_k = \omega_k \) for \( k = 0, 1, \ldots, N - 1 \) and \( \omega_0 = 2\pi/N \). The diagonal matrix \( \mathbf{S} \in \mathbb{C}^{N \times N} \) contains the DFT components of the signal, \( S(w_k) \), and \( \mathbf{E}_r \) denote the harmonic components for each delayed signal,

$$\mathbf{E}_r = [\mathbf{e}_{r_1} \ldots \mathbf{e}_{r_j} \ldots \mathbf{e}_{r_M}]$$

with \( \mathbf{e}_{r_j} = [1 e^{-j\omega \tau_j} \ldots e^{-j\omega(N-1)\tau_j}]^T \). Fading coefficients are arranged in vector \( \mathbf{h} \in \mathbb{R}^{M \times 1} \) and \( \mathbf{V} \in \mathbb{C}^{N \times 1} \) denotes noise samples.

For ranging applications, only the first delay needs to be estimated. To find the solution we separate the terms related to the the delay of interest \( \tau_j \) and the rest of delays terms are considered as part of the noise. Thus, (3) becomes,

$$\mathbf{Y} = \mathbf{Se}_{r_j} h_j + \mathbf{V}$$

The estimation problem resorts then to estimate the TOA from the frequency domain sampled signal (4). The proposed estimation algorithm is based on the observation that the problem is analogue to spectrum estimation. The motivation to consider a frequency domain approach is two fold: allows for lower complexity implementation requirements associated to the receiver architecture and enables accurate ranging estimation, which can potentially be implemented using high resolution spectral estimation algorithms such as the minimum variance (MV). Given the inherent high time resolution of the UWB signal we have considered simpler techniques, such as the periodogram. From classical spectral estimation theory it is known the periodogram provides an approximation to the signal spectrum in the form of [10],

$$\hat{P}(w_k) = |Y(w_k)|^2$$

where \( Y(e^{j\omega}) \) is the discrete Fourier transform of the data sequence. Interchanging the role of time-frequency variables, the temporal pseudo-spectrum proposed for estimation of TOA can be approximated by,

$$P(\tau) = \frac{e_{\tau}^* \text{Re}_{\tau_j}}{e_{\tau}^* e_{\tau_j}}$$

where \( \text{R} \) denotes the autocorrelation matrix of the sampled signal in the frequency domain.

IV. PROPOSED RANGING TECHNIQUES

The receiver architecture has a significant impact in the choice of the ranging technique. We focus on a receiver architecture based on a bank of analog filters, which allow for sub-Nyquist sampling rate [7] (Fig. 1 depicts a block diagram of the receiver architecture). The bank of filters are designed such that they form an orthogonal basis of the discrete time signal space which allows for an analog frequency-domain sampling of the receive signal. The sampled signal at the output of the bank of filters can be directly use, after AD conversion, as the input observation signal to the TOA estimator. The number of filters is a design parameter, which determines the estimator dimension, and is directly related to the sampling rate and the acquisition time of the AD converters at which the receiver operates.

![Fig. 1. Receiver block diagram.](image)

Next, we explicitly express (1) in terms of the repeated transmitted pulses for each symbol,

$$y(t) = \sum_{p=0}^{N_p-1} \sum_{m=1}^{M} h_m \sqrt{E_p} p(t - pT_{pr} - \tau_m) + v(t)$$

which in the frequency domain has the form,

$$Y(w) = \sum_{p=0}^{N_p-1} \sum_{m=1}^{M} h_m \sqrt{E_p} P(w)e^{-j\omega pT_{pr}}e^{-j\omega \tau_m} + V(w)$$

Rearranging into matrix notation the sampled received signal can be written as,

$$\mathbf{Y} = \sum_{p=0}^{N_p-1} \mathbf{Pe}_{r_j} a_j + \mathbf{V}$$

where \( \mathbf{P} \) is a diagonal matrix that contains the DFT components of the received pulse waveform shifted by a frequency factor \( e^{-j\omega pT_{pr}} \).
The explicit signal representation in terms of the repeated pulses allow us to describe better how to handle the temporal diversity of the IR-UWB signal. We have considered two ways of exploiting the temporal diversity of the UWB signal. The first approach considers each of the received pulses independently, from which a TOA estimate is obtained, followed by a diversity combining scheme, as illustrated in Fig. 2. The diversity combining scheme selects the minimum of the $N_p$ estimated TOAs [11]. The second approach, depicted in Fig. 3, combines the $N_p$ pulses to conform a better estimate of the covariance matrix $R_{YY}$, followed by the TOA estimation algorithm.

\[
\begin{align*}
    &Y^{(1)} \rightarrow \text{Estimation of} \quad \text{correlation} \\
    &\quad \text{matrix} \rightarrow \text{R}_{YY}^{(1)} \rightarrow \text{TOA} \\
    &\quad \text{estimation} \rightarrow \tilde{\tau}_0^{(1)} \\
    &\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \\
    &Y^{(N_p)} \rightarrow \text{Estimation of} \quad \text{correlation} \\
    &\quad \text{matrix} \rightarrow \text{R}_{YY}^{(N_p)} \rightarrow \text{TOA} \\
    &\quad \text{estimation} \rightarrow \tilde{\tau}_0^{(N_p)} \\
\end{align*}
\]

Fig. 2. Diversity schemes for ranging in IR-UWB: diversity combining

\[
\begin{align*}
    &Y^{(1)} \rightarrow \text{Estimation of} \quad \text{correlation} \\
    &\quad \text{matrix} \rightarrow \text{R}_{YY} \\
    &\quad \text{TOA} \\
    &\quad \text{estimation} \rightarrow \tilde{\tau}_0 \\
    &\vdots \quad \vdots \quad \vdots \quad \vdots \\
    &Y^{(N_p)} \rightarrow \text{Estimation of} \quad \text{correlation} \\
    &\quad \text{matrix} \rightarrow \text{R}_{YY}^{(N_p)} \\
    &\quad \text{TOA} \\
    &\quad \text{estimation} \rightarrow \tilde{\tau}_0^{(N_p)} \\
\end{align*}
\]

Fig. 3. Diversity schemes for ranging in IR-UWB: diversity used for covariance matrix estimation.

The TOA estimate is determined as the first $\tau$ in (5) that exceeds a given threshold. Instead of evaluating the pseudo-spectrum at each point over which the search is performed, we propose an alternative algorithm, namely root-periodogram, which finds the roots of a polynomial, thus reducing the search over a few points.

The idea is to find the maximum points of the pseudo-spectrum,

\[
\max \{ e^{H}_\tau R e_\tau \}
\]

via calculation of the roots of a polynomial. By properties of the trace and quadratic form (8) is equivalent to maximize the trace of

\[
\max \text{trace} \left( R e_\tau e^{H}_\tau \right)
\]

Denoting $E_j = e_{\tau_j} e^{H}_{\tau_j}$ and $\rho = e^{-jw_0 \tau}$, the matrix $E_j$ can be written as

\[
E_j = \begin{bmatrix}
    1 & \rho^{-1} & \rho^{-2} & \ldots & \rho^{-(N-1)} \\
    \rho & 1 & \rho^{-1} & \ldots & \rho^{-(N-2)} \\
    \rho^2 & \rho & 1 & \ldots & \rho^{-(N-3)} \\
    \vdots & \vdots & \vdots & \ddots & \vdots \\
    \rho^{N-1} & \rho^{N-2} & \ldots & \rho^{-1} & 1
\end{bmatrix}
\]

and the trace can be expressed in terms of the following polynomial,

\[
\text{trace}(RE_j) = \sum_{l=0}^{N-1} \sum_{k=0}^{N-1} R_{l,k} \rho^{(k-l)}(9)
\]

The maximum of $P(\tau)$ correspond to the roots of the derivative of (9) given by,

\[
\frac{\partial}{\partial \tau_j} \text{trace}(RE_j) = -jw_0 \sum_{l=0}^{N-1} \sum_{k=0}^{N-1} R_{l,k} \rho^{(k-l)} e^{-j(k-l)w_0 \tau_j}
\]

Replacing $n = k - l$,

\[
\frac{\partial}{\partial \tau_j} \text{trace}(RE_j) = -jw_0 \sum_{n=1-N}^{N-1} nR_n e^{-jnw_0 \tau_j}
\]

where $R_n = \sum_{k=1}^{N} R(k - n, k)$. The estimation problem reduces then to find the roots of the following polynomial

\[
\sum_{n=1-N}^{N-1} nR_n \rho^n = 0
\]

and then identify the ones that corresponds to a maximum (by evaluating them in the second derivative). Note that the coefficients of the polynomial are the addition of the off-diagonal elements of the correlation matrix, weighted by its off-diagonal index.

V. PERFORMANCE RESULTS

The proposed ranging algorithm has been evaluated targeting low rate applications, over the IEEE 802.15.4a channel models [12]. Only a single symbol of $N_p$ frames or pulses is used for TOA estimation, i.e. the correlation matrix is estimated from a single snapshot, which allows for a fast ranging estimation. In our simulation scenario the signal bandwidth is of 2GHz, the pulse repetition period $T_{pr}$ = 100 ns.

Fig. 4 depicts the average ranging error versus the signal-to-noise ratio (SNR) for a Line-Of-Sight (LOS) office environment, with transmission rate $R_b = 200$ kbps. The continuous line denotes the mean error in meters whereas the dashed line depicts the mean plus the standard deviation of the ranging error in meters. The averages over 100 realizations are plotted for several ranging schemes: RC-RootP relates to the TOA estimation based on the root-periodogram where a diversity scheme described by Fig. 3 has been considered. The TOA in RootP-DC is estimated applying the root-periodogram algorithm to each frame within a symbol and then selecting the minimum estimated TOA (Fig. 2). Equivalently, RC-P and P-DC apply diversity schemes Fig. 3 and Fig. 2, respectively, when the TOA is estimated via serial evaluation of the pseudo-spectrum over possible signal delays. Results show that the root version of the TOA estimator achieves better performance for both diversity schemes.

For a LOS scenario, where the direct path arrives with sufficient signal quality with SNR > −2 dB, it is observed that temporal diversity is slightly better exploited, when TOA
estimation is performed over each single frame and latter applied a diversity combining scheme rather than using the redundancy over the $N_p$ frames to obtain a better estimate of the correlation matrix to which the TOA estimation algorithm is applied. This results are in agreement with the results observed in [9]. For SNR $< -2$ dB the two schemes are not comparable, because the diversity combining scheme failed to obtain reliable estimates. Hence, for low SNR’s the diversity scheme using diversity for the covariance matrix estimation turns into a more robust ranging estimation technique. Shown for reference in Fig. 5 is the performance of the ranging techniques in NLOS office scenario.

Increasing the transmission rate (frame period is fixed to 100ns) the number of frames decreases, hence for high SNR’s the two diversity schemes converge to the same performance. In Fig. 6, the ranging estimation error is depicted for several transmission rates at SNR = -10 dB and for the NLOS office scenario. Due to the low SNR, only the diversity used for the covariance matrix estimation scheme is illustrated. If a lower transmission rate is considered, RC-P and RC-RootP exploit temporal diversity and the ranging accuracy improves. The mean error rises for higher transmission rates.

VI. Conclusion

In this paper, we have proposed a novel scheme for TOA estimation based on a frequency domain approach for ranging applications in IR-UWB systems. The considered approach shall suit attractive receiver architectures from an implementation point of view that could operate at sub-Nyquist sampling rate. We have introduced the TOA estimator based on a periodogram estimator and have improved the algorithm by reducing the search over the roots of a polynomial which coefficients can be easily found from the estimated correlation matrix. The resulting ranging scheme can operate with a single symbol observation interval. Note that the TOA estimator does not require a matrix inversion as compared to higher resolution methods such as the normalized minimum variance (NMV) estimator. This avoids estimation problems from ill-conditioned correlation matrices which allows for a reduced estimation time technique. Furthermore, two diversity schemes are investigated in combination with the TOA estimation algorithm and are evaluated over realistic channel models. We have demonstrated that for low SNR’s the diversity used for covariance matrix estimation scheme is more robust than the diversity combining scheme. For high SNR’s, the two schemes show nearly the same performance.
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